# Methodology of an Event-Driven Monte Carlo Missile Simulation

by

Mary Robin Holliday



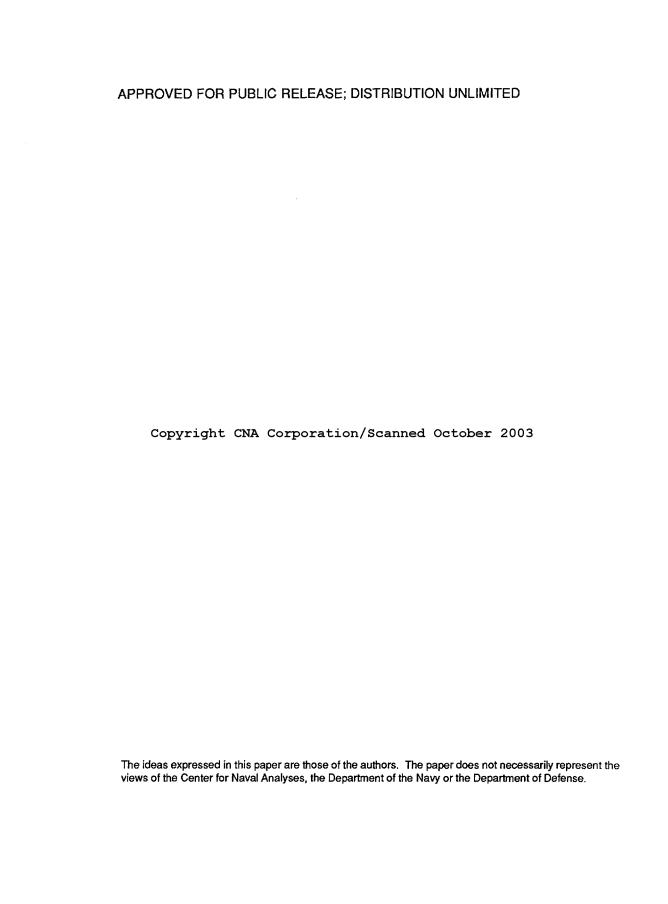
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# **Methodology of an Event-Driven Monte Carlo Missile Simulation**

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Mary Robin Holliday

Operations and Support Division



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#### METHODOLOGY OF AN EVENT-DRIVEN MONTE CARLO MISSILE SIMULATION

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Abstract. This paper describes the methodology used by the Center for Naval Analyses to develop a Monte Carlo missile simulation that computes probabilities of target acquisition and distributions of missile arrival times. The simulation accounts for numerous missile and target uncertainties. Coordinated strikes are simulated by modeling multiple missiles from multiple launch points firing on a group of targets. Tactical applications of the simulation required that it be computationally efficient. This led to an event-driven time advancement scheme.

#### INTRODUCTION AND OVERVIEW

The Center for Naval Analyses had developed a Monte Carlo missile simulation that computes probabilities of target acquisition (PACQA) and distributions of missile arrival times. The simulation accounts for numerous missile and target uncertainties. Targets may have independent motion or correlated motion and are permitted to vary direction and speed. A Random Tour motion model is used to determine the times a target changes direction. Coordinated strikes are simulated by modeling multiple missiles from multiple launch points firing on a group of targets. Each missile may have a unique combination of flight path and search mode and may be launched at any specified time during the engagement.

Tactical applications of the simulation include engagement planning and determination of salvo size based on probabilities of acquisition. Distributions of missile arrival times can aid in planning coordinated strikes and in indicating if target defenses can be saturated.

The simulation is event driven. Parameters are sampled each Monte Carlo iteration, and the times of their occurrences are determined. Critical events are those whose times of occurrence determine time steps in the simulation. Each such time is defined as a time of event (TOE). All TOEs are sorted in increasing order and the differences between successive TOEs are used as the time steps for a Monte Carlo iteration.

Once missile and target parameters are known (by Monte Carlo sampling) and the TOEs are determined, it is possible to determine which target, if any, would be acquired. A transcendental expression relating target and missile motion is used to determine the times when targets move into the seeker swath and their corresponding locations. If multiple targets meet the detection criteria, then the missile seeker is modeled to determine which target was acquired first. An iteration is complete when all missiles either acquire a target or complete their search unsuccessfully. This event-driven time advancement allows the simulation to maintain fidelity and still be computationally efficient.

#### SIMULATION APPLICATIONS:

Consider the scenario in figure 1. The launch point is defined as the geographic location from which N missiles are launched. The intended target is the unit which a missile is launched to acquire. The Area of Uncertainty (AOU) expresses the uncertainty in target location. Target positions are assumed to be distributed according to a bivariate normal distribution, typically a 90 percent containment ellipse. The AOU may reflect both errors in sensor localization and effects of target motion. The objective of the scenario is to conduct a coordinated missile engagement against the intended targets. In figure 1, the dashed lines represent the missile flight trajectories. The "springs" between targets indicate correlated target motion where the lead unit is located inside the hexagon.

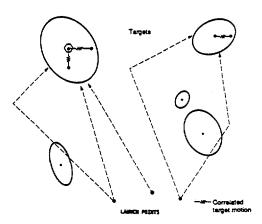


FIG. 1. Schematic of a coordinated engagement

The simulation computes the Probability of target Acquisition (PACQA) and missile arrival-time distributions. These distributions are calculated for a given missile acquiring a given target as well as for multiple missiles acquiring a common target. PACQA is calculated while considering all units near the target: hostile, neutral, and friendly. The probability of acquiring each unit in the area is determined (background units as well as the intended target). PACQA

is not the probability the missile will hit or kill the target. Specifically, PACQA does not consider missile reliability, survivability, or lethality.

The time each missile arrives on a target makes up one point of a notional arrival-time distribution. The arrival-time distributions are reported by means of bars whose widths represent time intervals and whose height are proportional to the total number of arrivals associated with those times. Therefore, an arrival-time distribution can aid in determining the most likely time the missile will arrive on target. Arrival-time distributions can also be generated for multiple missiles arriving on a common target and to indicate if it is possible to saturate target defenses. For example, let  $\Delta T$  represent the time interval to achieve saturation of target defenses. Let K represent the number of missiles that must arrive within time interval  $\Delta T$  to achieve saturation. It is therefore possible to predict if this saturation occurs and the most likely time of occurrence.

#### ASSUMPTIONS AND DESIGN CONSTRAINTS

The simulation was designed to be used as a "real-time" decision aid by the tactical commander located at the launch point. Input parameters are entered by an operator and the simulation output is required within several minutes. Therefore, the simulation was designed to be computationally efficient. In addition, the simulation must run on a desk-top computer since all launch points may not have large computer facilities. Also, the simulation must capture the physics of missile and target motion and their associated uncertainty with as much fidelity as possible given the first two constraints. The PACQAs and missile arrival-time distributions calculated by the simulation must be of high enough quality to assist a tactical decision-maker.

The simulation captures the physics of the problem by using the actual missile flight program to generate a nominal search plan (algorithm specific to a given missile type). Monte Carlo is used to account for missile and target uncertainties. Since targets rarely have constant direction and speed, Random Tour is used to model target motion. The simulation maintains speed by using an event-driven methodology and computationally efficient sorting and sampling techniques. In addition, the solution technique reduces the number of unknowns such that only a transcendental expression is solved each Monte Carlo iteration. The simulation was developed on a VAX and is currently being transferred to a desk-top computer. The simulation requires only a random-number generator.

#### SIMULATION STRUCTURE

The simulation can be divided into six major sections, each of which defines a computational task (see table 1). The input parameters to the simulation and the determined nominal missile behavior are constant throughout an engagement; therefore, these "front-end" calculations occur only once. In the latter four sections of the simulation, the sampling of missile and target parameters and the subsequent determination of PACQA are performed every iteration. Discussions of each of the six sections follow.

#### TABLE 1 Simulation structure

Calculations performed once	<ol> <li>Input parameters</li> <li>Design of the missile search plan</li> </ol>			
Calculations performed every iteration Time	Monte Carlo sampling of missi and target parameters     Definition of Critical Events			
neration Time	5. Determination of Intercept Time 6. Target acquisition			

#### **Input Parameters**

The input parameters of the simulation are given in table 2. These input parameters describe the overall engagement—that is, the number of launch points, corresponding salvo size, and intended target. More detailed information is required to plan the flight trajectory a missile will use to search for its intended target. In table 2, a target's radar cross section (RCS) is reflected through the target's detection range. A target must be located in the missile's search area and seeker swath as well as within its detection range in order to be acquired. The detection range of a target is also a function of environmental conditions and must be determined off-line and input to the simulation.

#### TABLE 2 Simulation input parameters

- 1. Location of each launch point (Lat, Long)
- 2. Launch point salvo size
- Launch point environmental conditions (wind, temperature, rain)
- 4. Intended target for each missile
- 5. Search mode for each missile
- Flight trajectory for each missile (initial heading, flyout waypoints)
- For each target: a location (AOU size and orientation), direction, speed and associated uncertainty, time of report
  - Correlated target motion: information required for lead unit only. Uncertainties about formation are input for other units.
- 8. Target detection range (n.mi)

Location, direction, speed, and their associated uncertainties are required for each target. Also, the time-late for this information must be reported. However, the source of the targeting information is irrelevant. For example, the information can come from either a tracking algorithm or from a single-position report.

#### Design of the Missile Search Plan

The missile search plan was determined by coding a portion of the missile's Command Launch Software (CLS) into the simulation. The algorithm coded is specific to a given missile type. The CLS determines the missile flight trajectory based on input parameters as well as the missile search area (area in which missile seeker will accept returns).

#### Monte Carlo Sampling of Parameters

The simulation accounts for both missile and target uncertainties. Missile uncertainties are characteristic of the missile engineering design and environmental conditions. The target uncertainties typically are scenario dependent and are treated as input parameters.

Figure 2 illustrates some of the missile and target uncertainties included in the simulation. The straight solid line shows the planned flight trajectory for a single missile. The "fork" in the path indicates the position at which seeker initiation should occur; the area covered by the seeker (i.e., the area between the parallel solid lines) indicates the planned area to be searched by the missile.

The launch point may not have perfect navigation, as illustrated by the small circle at the base of figure 2; therefore, the launch point actually may be several miles from the estimate. This error could be important, because the designed missile flight trajectory is based on target and launch point latitudes and longitudes. The bearing and range to the target is calculated from this information. As is shown in the figure, launch point location error may cause the missile to search a different geographic location relative to the target position than was intended. A missile flight trajectory and a search area consistent with such an error are illustrated by the dashed lines in the figure.

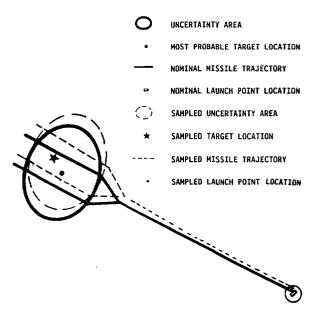


FIG. 2. Monte Carlo sampled missile and target parameters

Uncertainty in the location of a target is expressed in terms of an AOU. Target positions are assumed to be distributed according to a bivariate normal distribution. The AOU is typically the 90-percent containment ellipse for such a distribution. An AOU may reflect both errors in sensor reporting and the effects of target motion.

The AOU expands with time to account for target motion. The designed missile search area considers target motion as well as missile navigation errors. The missile search area is illustrated in figure 2 by the large solid elliptical shape. The planned geographic location of the missile search area is less important than the area actually searched. For example, if a missile drifts off course 8 miles during flyout, the search area will be shifted 8 miles. This is illustrated in figure 2 by the dashed elliptical shape.

When the missile search pattern is designed, the nominal flight trajectory is determined. To account for uncertainty in missile behavior, missile parameters are sampled in each Monte Carlo iteration. Missile heading bias, velocity uncertainty, uncertainty in the missile flight trajectory due to wind, and azimuthal drift are all modeled and are assumed to be normally distributed.

Target location, direction, speed and corresponding uncertainty are scenario dependent and are treated as input parameters. The AOU is used to obtain the target location input parameters. The AOU is an ellipse whose center is the best point estimate (BPE) of the target location. The user specifies the probability that the target is in the ellipse. The simulation computes the associated standard deviations, assuming the target location is characterized by a bivariate normal distribution. If independent target motion is assumed, then each target's location at the beginning of an iteration is determined by sampling from this distribution.

The simulation also accounts for correlated target motion. The initial location of the lead unit (LU) is sampled from its input AOU. Other units are located at their reported distance and bearing from the LU with some uncertainty to allow for station keeping. The variation in each unit's position relative to the LU is an input parameter. Direction and speed are determined for the LU only. All other units are moved relative to the LU, with some specified uncertainty to further allow for station keeping.

The simulation uses a Random Tour motion model to determine the times a target changes direction. Let  $(\lambda i)$  be defined as the expected number of direction changes made by target (i) in one hour.  $(\lambda i)$  is based on observed data for a given target type. Times between direction changes are independent and exponentially distributed with the mean time between direction changes equal to  $(1/\lambda i)$  of an hour. Target motion is linear between direction changes. Exponentially distributed random variables are generated by:

$$S_k = -\frac{1}{\lambda_i} \left[ \ln \left( 1 - x_k \right) \right] ,$$

where

 $S_k$  = times between direction changes

 $x_k$  = uniformly distributed random number.

The actual direction and speed of a target are determined based on the quality of targeting information, and the operator's perception of future target motion. All sampling of target parameters is done in a target's own coordinate system, where the most probable target location is defined as the origin. When determining if a target lies within a missile's field of view, the target coordinates are translated to the missile coordinate system for that calculation.

#### **Definition of Critical Events**

This simulation is event-driven. Parameters are sampled in each Monte Carlo iteration, and the corresponding times of their occurrence are determined. Critical events are those whose times of occurrence determine time steps in the simulation. Each such time is defined as a time of event (TOE).

A TOE occurs whenever a missile or target deviates from its previous mode of operation. The only TOE associated with a target is the time of a direction change. Several missile-related events generate TOEs. Missile TOEs include the time the missile seeker turns on, the time the missile begins and completes searching one flight leg, and the time the missile seeker turns off. The missile flight trajectory is assumed to be linear between its TOEs.

All TOEs are sorted in increasing order, and the differences between successive TOEs are used as the time steps for a Monte Carlo iteration. This event-driven time advancement allows the simulation to maintain fidelity and still be computationally efficient. An iteration is complete when all missiles either acquire a target or complete their search unsuccessfully. When using the maximum number of missiles and targets, as many as 800 times must be sorted per Monte Carlo iteration; therefore, a computationally efficient sorting routine was required. The shell-sort algorithm described later is used in the simulation.

#### Determination of Intercept Time

Both missile and target motion are linear between TOEs. The range at which a given target can be detected is a user input. This range is limited by the maximum detection range of the missile seeker and is a function of a target's RCS. The missile and target motion and the detection range can be related in one expression in which the only unknown is the time of intercept, that is, the time at which the target first moves into the missile's field of view.

Each target's position and heading are translated in each time step from the target coordinate system to each missile's coordinate system. The target coordinate system is defined with the origin located at the most probable target location, and the axes are aligned with the AOU orientation as illustrated in figure 3. YTAR and XTAR

define the target centered coordinate system. The angle  $(\beta)$  is defined as the orientation form north of the target-centered coordinate system.

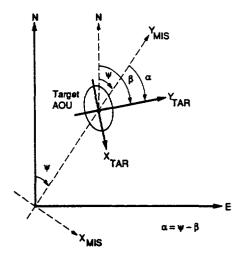


FIG. 3. Target and missile coordinate systems

YMIS and XMIS define the missile coordinate system whose origin is located at the launch point. A direct line from the launch point to the most probable target location initially defines the missile coordinate system as given in figure 3. The angle  $(\psi)$  is defined as the orientation from north of the missile-centered coordinate system. The angular difference between the missile and target coordinate systems is defined as  $(\alpha)$ . The definitions of additional terms are given in table 3. The remainder of this section develops the expressions used to determine time of intercept. The objective is to find the time when the missile and target first close to the specified detection range L, as illustrated in figure 4.

A target's location can be expressed in a missile's coordinate system by the expression:

$$Y_T = Y \cos(\alpha) + X \sin(\alpha) + Y_k ,$$
  

$$X_T = -Y \sin(\alpha) + X \cos(\alpha) + X_k ,$$

Target and missile motion as a function of time can be expressed as:

$$\begin{split} X_T &(t) = X_{To} + tV_T F_X &(\Theta_T) \\ Y_T &(t) = Y_{To} + tV_T F_Y &(\Theta_T) \\ X_M &(t) = X_{Mo} + tV_M F_X &(\Theta_M) \\ Y_M &(t) = Y_{Mo} + tV_M F_Y &(\Theta_M) \end{array} ,$$

As illustrated in figure 4, the projective distances between missile and target can be expressed by the following equations:

$$\frac{X_T(t) - X_M(t)}{L} = \cos \psi , \quad \frac{Y_T(t) - Y_M(t)}{L} = \sin \psi .$$

TABLE 3 Variable Definition for Determination of Intercept Time

	Interc	ept Time
X , Y	=	x and y coordinates of the target in the target-centered coordinate system
$X_T$ , $Y_T$	=	x and y coordinates of the target in the missile-centered coordinate system
$X_k$ , $Y_k$	=	projection of the target coordinates in the target-centered coordinate system onto the x and y axis, respectively, of the missile-centered coordinate system
$X_M$ , $Y_M$	=	x and $y$ coordinates of the missile in the missile coordinate system (n.mi.)
$V_T$	=	velocity of target (kt)
$V_{M}$	=	velocity of missile (kt)
$\Theta_T$	=	heading of target in missile coordinate system (rad)
$\Theta_{M}$	=	heading of missile in missile coordinate system (rad)
<i>L</i>	=	detection range of missile for a given target (n.mi.)
$X_{To}$ , $Y_{To}$	=	x and y coordinates of the target in the missile coordinate system at the beginning of the time interval
$X_{Mo}$ , $Y_{Mo}$	=	x and y coordinates of the missile in the missile coordinate system at the beginning of the time interval
t	=	time (hours)
ť	=	intercept time, time when missile and target are distance L apart (hours)
$F_X(\Theta), F_Y(\Theta)$	)) =	expressions that determine the correct trigonometric function to model target motion where $\theta=\theta_T$ , or missile motion where $\theta=\theta_M$ .

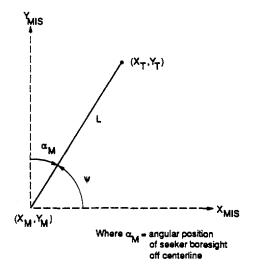


FIG 4. Target-missile relative positions

For each quadrant, there is an angle  $\psi$  for which the above expressions are satisifed. Let t' be the intercept time. Substitute in the expressions for target and missile motion where t = t'.

$$L \cos \Psi = X_{To} + t'V_T F_X (\Theta_T) - X_{Mo} - t'V_M F_X (\Theta_M)$$
  
$$L \sin \Psi = Y_{To} + t'V_T F_Y (\Theta_T) - Y_{MO} - t'V_M F_Y (\Theta_M) .$$

Define the differences in initial missile and target position:

$$\Delta X = X_{To} - X_{Mo}$$
$$\Delta Y = Y_{To} - Y_{Mo} .$$

Define differences in relative velocity as

$$\begin{split} V_X &= V_T F_X(\Theta_T) - V_M F_X(\Theta_M) \\ V_Y &= V_T F_Y(\Theta_T) - V_M F_Y(\Theta_M) \end{split} \ . \end{split}$$

Therefore, the expression for relative motion can be simplified to

$$L \cos \Psi = \Delta X + t'V_X$$
$$L \sin \Psi = \Delta Y + t'V_Y$$

Square both sides of the equations and add the expressions together to obtain one expression for intercept time:

$$L^{2} = L^{2} \left(\cos^{2} \psi + \sin^{2} \psi\right) = \Delta X^{2} + \Delta Y^{2}$$
$$+ t' \left(2\Delta X V_{X} + 2\Delta Y V_{Y}\right) + t'^{2} \left(V_{X}^{2} + V_{Y}^{2}\right)$$

This is a quadratic in t' with all coefficients known. The quadratic expression will have no positive, real solution if the target is not located in the area searched by the missile. Also, the intercept time t' must fall within the current time interval (i.e., within two successive TOEs) to be valid.

#### Target Acquisition

A target is assumed to be detected if it lies within its corresponding detection range and is located in the seeker swath and missile search area at that corresponding time. In every time interval, t' is determined for each target and missile combination. Intercept time is determined for missiles only after they have begun their search. For a given missile, the smallest intercept time determined corresponds to the target that is intercepted first by that missile. The target intercepted first is not necessarily the target acquired, since the target must also be located in the seeker swath and missile search area at this time. For multiple targets whose intercept time falls within the time interval required for the missile to lock on target, the missile seeker is modeled in more detail.

Once a missile detects a target, time of arrival is determined assuming a straight-line trajectory from missile to target.

The target is assumed to continue moving after multiple missile hits (no mobility kill). An iteration is complete when all missiles either acquire a target or complete their search unsuccessfully.

#### SIMULATION COMPUTATIONAL EFFICIENCY

The simulation is a temporally event-driven model to achieve computational efficiency. In addition, options for several underlying processes in the simulation were examined for efficiency and accuracy and have been documented in [1] and [2].

Several techniques for generating normally distributed random numbers were examined for use in the simulation [1]. The polar technique, developed by Atkinson and Pearce, was chosen [3]. The polar technique is an improvement over the earlier-developed Box and Muller technique that generates normally distributed values given uniformly distributed values. Box and Muller maintained a one-to-one correspondence between the uniformly and normally distributed value but required the calculation of sine and cosine. The polar technique eliminates these trigonometric calculations, thereby increasing the computational efficiency of the algorithm. The polar technique uses two uniform random values to generate two normal random values; however, the technique rejects values with a probability of  $(1 - \pi/4)$  [3]. Even with the rejection of variables, the polar technique was found to be between 9 and 31 percent faster (depending on the machine) in the FORTRAN programming language than the Box and Muller technique [3], and at least twice as fast as techniques examined in reference [1].

The shell-sort algorithm was chosen for use in the simulation [2]. The shell-sort required about 4.7 CPU seconds to sort 20,000 uniformly distributed random values, which was more than an order of magnitude faster than the examined algorithms [2]. The shell-sort algorithm makes use of an interchange sort [4]. In the simplest application of an interchange sort, pairs of numbers are exchanged one step at a time until an array is sorted. It is important to note that the computational efficiency of an interchange sort depends entirely on the array values exchanged. The shell-sorts enhancement to the interchange sort is to allow array values to move greater distances at

first and then move smaller distances as their final destination is approached. The shell-sort is also referred to as the diminishing increment sort. Conceptually, the shell-sort is simple; however, the mathematical analysis to determine initial and diminishing array value movement (jump size) is complex. This work uses a straightforward application of the shell-sort. The initial jump size is chosen to be half the array length. The jump size is then repeatedly halved until the array is sorted. The shell-sort algorithm is programmed in about 20 lines of FORTRAN. This algorithm has the additional advantage that only the array itself and one other value are required to be stored.

The computational efficiency of this event-driven Monte Carlo missile simulation is best illustrated by example. Consider the scenario in which two launch points each fire one missile at their intended targets. There are a total of six independently moving targets being modeled. The simulation, run for 1,000 Monte Carlo iterations on a VAX 11/8650 (with I/O), required 24 CPU seconds.

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